MIPP TPC Data Reconstruction

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1 Introduction

The centerpiece of the MIPP experiment is the EOS TPC. This note explains how the data are reconstructed offline to provide both tracking and particle identification.

2 TPC Physical Parameters

The EOS TPC is a rectangular box with a drift volume enclosed by a field cage on the sides and a pad plane (PWC) on the bottom. The drift volume has dimensions $(z \times x \times y)$ 150 cm $\times$ 96 cm $\times$ 75 cm, and is typically operated with 90% Ar + 10% CH_4 at one atmosphere pressure. The pad plane consists of a solid array of 1.2 cm $\times$ 0.8 cm pads (15360 pads in total). There are 120 pad columns (in x) and 128 pad rows (in z). Data are sampled at 10 MHz. In the MIPP experiment, the TPC operates inside a dipole magnetic field (inside the Jolly Green Giant (JGG)) parallel to the electric field.

The right-handed Cartesian coordinate system we will use in the TPC reconstruction has the positive z-axis pointing along the direction of the beam, the positive y-axis pointing upward and the positive x-axis pointing left (if one faces downstream).

3 TPC Raw Data Format

The data from the TPC is packaged into packets; one for each quadrant for each event. For the full TPC that is 4 packets per event. A MIPP event is composed of many packets each starting with a 3 word header. The header for every packet is of the form:

- PacketID (4 bytes)

- PacketVersion (4 bytes)

- PacketSize (4bytes)

The Packet ID specifies where the data is from. Each detector/data source has a unique packet ID. The TPC has 4 packet ID's in any given event. The packet version is just a versioning that allows one to change the format and keep forward and backward compatability. The Packet size is a count of the number of bytes in the packet including the 3 word header (12bytes).

Currently (24feb04) there are only two types of TPC packet IDs, compressed and uncompressed. For each type there are 4 ID that correspond to the four quadrants. The quadrant number is specified by the hardware, and is fixed. In other words the quadrant specifies physically where the data is comming from. The packet IDs are specified in the MIPP IO software and can be found in MippIoBlock.h.

3.1   Uncompressed Data Format

The Packet IDs for that uncompressed data format of the four TPC Quadrants are
```
kTPCQuad0UncompressedId = 0xE90703D0,
kTPCQuad1UncompressedId = 0xE90703D1,
kTPCQuad2UncompressedId = 0xE90703D2,
kTPCQuad3UncompressedId = 0xE90703D3
```
The uncompressed format contains all of the avliable data in the TPC. In the case of 256 timebucket setting this is 120x128x256 data points or just under 4 million adc values for the full TPC. This format is fixed in size from event to event since the data volume does not depend on the multiplicty of the event.

Within a TPC packet each stick is written with a header and followed by data. There is no specified sequence of how the sticks are written and if a stick is not configured it is simply not written in the packet. The event is read by simply reading sticks from the data stream until the end of the packet is reached. The sticks all have the same format within this packet and is described below.

3.2   Compressed Data Format

The Packet IDs for that *compressed* (zero-supressed) data format of the four TPC Quadrants are

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Uncompressed Data Sub-Packet Stick Format		
	Num. Bytes	Offset
Number of adc values	2	0x00
DSP processing mode	2	0x02
DSP event counter	2	0x04
DSP recording threshold	2	0x06
STICK location	1	0x17
STICK serial number	1	0x19
STICK house keeping	0x10	0x20
start of STICK data		0x200

Table 1: Stick format for the TPC uncompressed raw data format.

```
kTPCQuad0CompressedId = 0xE90703C0,
kTPCQuad1CompressedId = 0xE90703C1,
kTPCQuad2CompressedId = 0xE90703C2,
kTPCQuad3CompressedId = 0xE90703C3
```

The format of the header is the same as for uncompressed data. The data starts at an offset of 0x20. A magic number is located at this offset of 0x8888. Compressed data follows and the end is terminated with another magic number 0x9999.

The compressed data uses the most significant nibble of the adc 2 byte word to signify what data is contained in the word. The possible values are:

- 0 = normal adc value

- 8 = stick id

- b = pad id

- e = bucket number

The data is pedestal subtracted and the data that is above a threshold is recorded. The data is usually contigious for some number of buckets so the stick,pad and bucket numbers are not written for each adc in a cluster. The adc values are assumed to be contigious when stick,pad,bucket is not given. An example of this would be:
stick number

```
pad number
bucket number
adc value
adc value
adc value
bucket number
adc value
pad number
bucket number
adc value
etc...
```

3.3   TPC Pedestal Data

3.4   TPC Event Data

4   TPC Hit Reconstruction

As a charged particle passed through the TPC, the gas is ionized and the electrons drift downward toward the TPC anodes and pad plane. The gas ionization is in the form of an electron cloud that extends a few cm in both x and y, therefore covers several pad widths and time buckets. It is this cloud, which we refer to as a "cluster" that serves as the basis of the TPC reconstruction.

4.1   Cluster Finding

The raw TPC digits are first sorted by pad row (z). In each pad row, 2-dimensional clusters are formed by recursively searching for vertically, horizontally and diagonally connected digits.

The very first event of a run contains the uncompressed TPC pedestal data. Dead (hot) pads are masked-off by having a pedestal value of 0xfff. In the TPC cluster reconstruction, if event 0 is processed a 3-dimensional map is formed of dead pads. When forming 2-D clusters, dead pads are ignored and the algorithm continues to look in the same direction for neighboring digits. The number of times dead pads are ignored in any single direction is typically set to three (this parameter is configurable via XML).

4.2   Hit Finding

4.2.1   Peak Finding

Often there are multiple hits in a single cluster; this occurs when tracks are close to each other, either because of pileup or because they are close to the vertex position. We therefore attempt to form individual hits from the cluster by searching for multiple peaks. Currently, we only search for peaks in the time (y) direction. A "peak" is defined as a collection of ADC values, ordered in time, on a single pad where:

1. $q_i > q_{thresh}$ (q_{thresh} is to-be-determined)

2. a "FoundPeak" flag is thrown if $q_i < q_{max} - 2\sqrt{q_{max}}$

3. if (FoundPeak == true && $q_i < q_{min}$) then $q_{min} = q_i$

4. if (FoundPeak == true && $q_i < q_{thresh}$), we store the peak and begin searching for a new peak

5. if (FoundPeak == true && $q_i > q_{min}$), we store the peak and begin searching for a new peak

6. a peak has at least two time buckets

7. a peak has $q_{max}/q_{min} > 2$

If a peak has four or more ADC values, we next try to fit the time distribution of the peak to a Gamma distribution function as described in the next section.

4.2.2   Peak Fitting: The Linear Least Squares Method

The shape of a cluster is typically Gaussian in the x-direction, whereas in time (y-direction) the shape follows the Gamma distribution function due to the pulse-shaping electronics of the ADCs. TPC hit positions were initially defined as the center-of-gravity of the cluster, and the uncertainty taken as the RMS of the distribution (in both the x and y directions. However in this approach, threshold effects significantly alter the position and measured integrated charge of the cluster. A more accurate position and dE estimation

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may be determined by fitting the clusters to a Gamma distribution function along the time axis. The charge distribution for a single pad has the form:

$$q(t) = A \left(\frac{t - t_0}{\tau}\right)^{\gamma - 1} \exp\left[-\left(\frac{t - t_0}{\tau}\right)\right] \qquad (1)$$

where the parameters τ and γ are determined from the TPC electronics.

To fit the time distribution of ADC values for a single pad, we wish to minimize the function

$$\chi^2 = \sum_{i=0}^{n} [\ln q_i - \ln q(t_i)]^2 w_i \qquad (2)$$

where

$$\ln q(t_i) = \ln A + (\gamma - 1)\ln(t - t_0) - (\gamma - 1)\ln\tau - \frac{t - t_0}{\tau} \qquad (3)$$

To make χ^2 a linear function of fit parameters, we fix t_0 and define $\Delta t_i = t_i - t_0$. Once the other three parameters of the Gamma distribution are determined by minimization of the χ^2, t_0 may be determied to sufficient precision by finding the value of t_0 which gives the smallest value of χ^2. We have found that if a good first guess of t_0 is made, a simple linear search works quite well.

Letting $\kappa = \gamma - 1$, minimization of χ^2 gives

$$\frac{\partial \chi^2}{\partial A} = \sum \left[\ln q_i - \ln A - \kappa \ln \Delta t_i + \kappa \ln \tau + \frac{\Delta t_i}{\tau}\right] w_i = 0 \qquad (4)$$

$$\frac{\partial \chi^2}{\partial \tau} = \sum \left[\ln q_i - \ln A - \kappa \ln \Delta t_i + \kappa \ln \tau + \frac{\Delta t_i}{\tau}\right] \times \left(\kappa - \frac{\Delta t_i}{\tau}\right) w_i = 0 \qquad (5)$$

$$\frac{\partial \chi^2}{\partial \gamma} = \sum \left[\ln q_i - \ln A - \kappa \ln \Delta t_i + \kappa \ln \tau + \frac{\Delta t_i}{\tau}\right] \times (\ln \Delta t_i - \ln \tau) w_i = 0 \qquad (6)$$

From Eq. 4 we see that

$$\sum w_i \ln q_i = (\ln A - \kappa \ln \tau) \sum w_i + \kappa \sum w_i \ln \Delta t_i - \frac{1}{\tau} \sum w_i \Delta t_i \qquad (7)$$

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From Eqs. 5 and 7 we find:

$$\sum w_i \Delta t_i \ln q_i = (\ln A - \kappa \ln \tau) \sum w_i \Delta t_i +$$
$$\kappa \sum w_i \Delta t_i \ln \Delta t_i - \frac{1}{\tau} \sum w_i \Delta^2 t_i \qquad (8)$$

And from Eqs. 6 and 7 we find:

$$\sum w_i \ln q_i \ln \Delta t_i = (\ln A - \kappa \ln \tau) \sum w_i \ln \Delta t_i +$$
$$\kappa \sum w_i \ln^2 \Delta t_i - \frac{1}{\tau} \sum w_i \Delta t_i \ln \Delta t_i \qquad (9)$$

Equations 7-9 can be rewritten in matrix form as

$$\vec{b} = \mathbf{C}\vec{a} \qquad (10)$$

where

$$\vec{a} = \begin{pmatrix} (\ln A - \kappa \ln \tau) \\ -1/\tau \\ \kappa \end{pmatrix}, \qquad \vec{b} = \begin{pmatrix} \sum w_i \ln q_i \\ \sum w_i \Delta t_i \ln q_i \\ \sum w_i \ln \Delta t_i \ln q_i \end{pmatrix},$$

and

$$\mathbf{C} = \begin{pmatrix} \sum w_i & \sum w_i \Delta t_i & \sum w_i \ln \Delta t_i \\ & \sum w_i \Delta^2 t_i & \sum w_i \Delta t_i \ln \Delta t_i \\ & & \sum w_i \ln^2 \Delta t_i \end{pmatrix}$$

Therefore, $\vec{a} = \mathbf{C}^{-1}\vec{b}$ and

$$\kappa = \gamma - 1 = a_3, \qquad (11)$$
$$\tau = -1/a_2, \qquad (12)$$
$$A = \tau^\kappa \exp(a_1) \qquad (13)$$

Since

$$\delta^2 a_i = 2 \left(\frac{\partial^2 \chi^2}{\partial a_i^2} \right)^{-1}, \qquad (14)$$

we find

$$\frac{\partial \chi^2}{\partial A^2} = \frac{2}{A^2} \left[C_{11}(1 - \ln A + \kappa \ln \tau) + b_1 + \frac{1}{\tau} C_{12} - \kappa C_{13} \right] \qquad (15)$$

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$$\frac{\partial \chi^2}{\partial \tau^2} = 2[C_{11}\left(\frac{\kappa}{\tau^2}(\kappa + a_1)\right) - C_{01}\frac{2}{\tau^3}\left(\frac{3}{2}\kappa + a_1\right) +$$
$$\frac{1}{\tau^2}\left(C_{13} + \frac{1}{\tau^2}C_{22} - \frac{2K}{\tau}C_{23} + \frac{2}{\tau^2}C_{33} - \kappa b_1 + \frac{2b_2}{\tau}\right)] \quad (16)$$

and

$$\frac{\partial \chi^2}{\partial \kappa^2} = 2[\ln^2 \tau C_{11} - \ln \tau C_{13} + C_{22}] \quad (17)$$

Furthermore, χ^2 can be rewritten as

$$\chi^2 = \sum w_i \ln^2 q_i - 2\vec{a} \cdot \vec{b} + \sum a_i^2 C_{ii} + 2(a_1 a_2 C_{12} + a_1 a_3 C_{13} + a_2 a_3 C_{23}) \quad (18)$$

Finally, the integrated charge (area under the fit curve) is given as

$$Q_{total} = A\tau\Gamma(\kappa + 1) \quad (19)$$

An uncertainty on this is easily found as

$$\delta^2 Q_{total} = Q^2 \left[\left(\frac{\delta A}{A}\right)^2 + \left(\frac{\delta \tau}{\tau}\right)^2 + \left(\frac{\delta \kappa}{\Gamma(\kappa + 1)}\right)^2 \left(\frac{\partial \Gamma(\kappa + 1)}{\partial \kappa}\right)^2\right] \quad (20)$$

Here, the derivative $\partial\Gamma(\kappa+1)/\partial\kappa$ is calculated numerically. For those peaks that were not fit, the total charge is taken as the sum of ADC values in the peak.

4.2.3 Sample Weights

To determine the best weights to use in the χ^2 minimization described in Section 4.2.2, we produced 10000 Gamma distributions with Poisson uncertainties in each bin. The default of the parameters were $A = 150$, $\tau = 1$. and $\kappa = 2.5$ (t_0 was fixed to 0). Each Gamma distribution was then fit using the above method. Four different weighting schemes were used: $w_i = \sqrt{q_i}$, q_i, $0.5\ln(q_i)$ and q_i^2. By eye, using weights of $w_i = q_i^2$ appeared to give the best fit results (see Fig. 1). Furthermore, the distribution of fit residuals (Fig. 2) for the case where $w_i = q_i^2$ has the smallest RMS and is more symmetric than the distribution of fit residuals of the other three weighting schemes. We therefore follow the same approach used by the E910 experiment of using $w_i = q_i^2$.
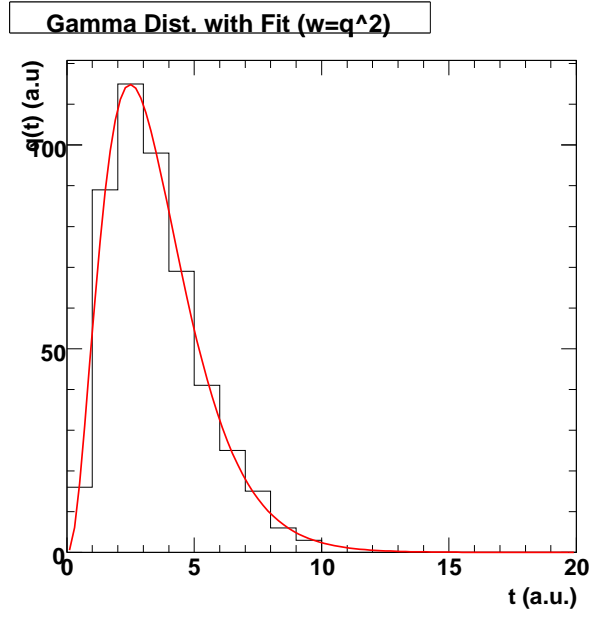
Figure 1: A single gamma distribution (with statistical uncertainties in each bin) fit using the method described in Section 4.2.2 and using $w_i = q^2$.

(a) $w_i = \sqrt{q_i}$ (b) $w_i = q_i$
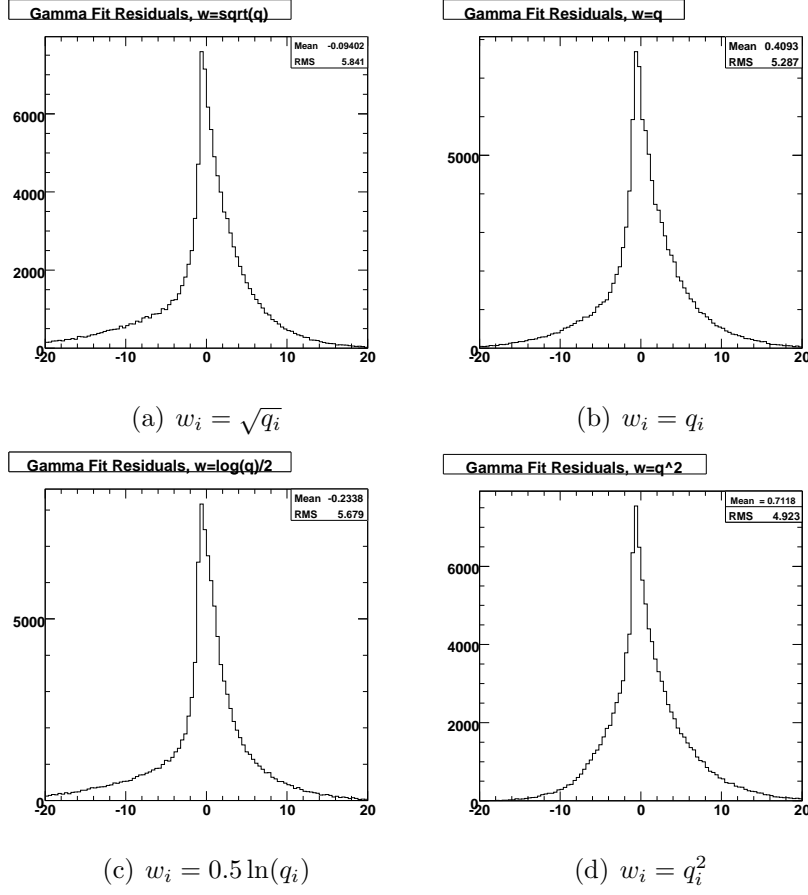
(c) $w_i = 0.5 \ln(q_i)$ (d) $w_i = q_i^2$

Figure 2: Gamma fit residuals using different weighting schemes. $w_i = q_i^2$ results in the narrowest residual distribution.

4.2.4 Peak Fitting Results

4.3 Hit Formation

Once the peaks are formed and fit, we next form hits by grouping peaks on neighboring pads that have a peak time within 2 buckets of each other. For those peaks that had an insufficient number of buckets to fit, or for those peaks where the fit failed, the peak position is taken as the weighted mean of the time distribution. Otherwise the peak time is taken as the most-probable-value (MPV) of the Gamma distribution function: $t_{peak} = t_0 + \tau\kappa$.

Hits are required to have at least two peaks from neighoring pads. How-

ever, if a good, clean peak is found in a single pad but no other obvious peaks on a neighboring pad, a hit is formed from that single peak. A "clean" peak is currently defined as a peak with $0 < \chi^2/\mathbf{ndf} < 30$, and $q_{max} > 100$.

4.3.1 Hit dE Determination

TPC hit dE information is stored in two variables, one that holds the sum of the integrated charge of peaks that fit to the Gamma distribution, and another that holds the sum of the ADC values of peaks that were not fit. One can then later decide how best to combine this information to get the best $\langle dE/dx \rangle$ determination.

4.3.2 Hit Position Determination

A pad-plane hit position is then determined as:

$$t_{hit} = \frac{\sum w_i t_{i_{peak}}}{\sum w_i} \tag{21}$$

$$c_{hit} = \frac{\sum w_i c_i}{\sum w_i} \tag{22}$$

where c_i is the pad column number, and w_i is either the integrated charge determined from the Gamma distribution fit or 10% of the sum of the ADCs in a peak that was not fit. A lesser weight is given to those peaks that were not fit since these peaks contain less position information.

The uncertainties of the hit positions are taken as

$$\sigma_t^2 = \frac{1}{N-1} \left[\frac{\sum w_i t_i^2}{\sum w_i} - t_{hit}^2 \right] \tag{23}$$

and

$$\sigma_c^2 = \frac{1}{N-1} \left[\frac{\sum w_i c_i^2}{\sum w_i} - c_{hit}^2 \right] \tag{24}$$

The initial (x, z) position of the TPC hit is obtained by transforming the average pad column and the pad row positions of the hit into global (eg, CAVE) coordinates on the pad plane of the TPC. The final (x, y, z) position is determined by drifting the hit from the pad plane back up into the gas-volume of the TPC. The drift distance is determined by t_{hit} and the drift-velocity in the TPC.

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4.4 Distortion Corrections

The equation of motion of a charged particle in both a magnetic and electric field with a friction term is

$$\frac{d\vec{u}}{dt} = \frac{e}{m}\vec{E} + \frac{e}{m}\vec{u} \times \vec{B} - \frac{\vec{u}}{\tau} = 0 \tag{25}$$

where $\vec{u}$ is the velocity, e is the charge, m is the mass of the particle, $\vec{E}$ is the electric field, $\vec{B}$ is the magnetic field and τ is the particle's mobility. We assume a steady-state solution to this equation ($d\vec{u}/dt = 0$), so that

$$\vec{u} - \frac{e}{m}\tau(\vec{u} \times \vec{B}) = \frac{e}{m}\tau\vec{E} \tag{26}$$

Working out the algebra, we find

$$\mathbf{M} \cdot \vec{u} = -\frac{\omega}{B}\vec{E} \tag{27}$$

where

$$\mathbf{M} = \begin{pmatrix} 1/\tau & \omega_z & -\omega_y \\ -\omega_z & 1/\tau & \omega_x \\ \omega_y & -\omega_x & 1/\tau \end{pmatrix}$$

and $\omega_i = \frac{e}{m}B_i$ and $\omega^2 = \omega_x^2 + \omega_y^2 + \omega_z^2+$. Note that the signs of the ω terms in already take into account the charge of the electron[1].

Inverting the matrix $\mathbf{M}$, we find

$$\mathbf{M}^{-1} = \frac{\tau}{1 + \omega^2\tau^2} \begin{pmatrix} 1 + \omega_x^2\tau^2 & -\omega_z\tau + \omega_x\omega_y\tau^2 & \omega_y\tau + \omega_x\omega_z\tau^2 \\ \omega_z\tau + \omega_x\omega_y\tau^2 & 1 + \omega_y^2\tau^2 & -\omega_x\tau + \omega_y\omega_z\tau^2 \\ -\omega_y\tau + \omega_x\omega_z\tau^2 & \omega_x\tau + \omega_y\omega_z\tau^2 & 1 + \omega_z^2\tau^2 \end{pmatrix}$$

In MIPP, $\vec{E} = (0, E_y, 0)$, $|E| = |E_y|$ and $\vec{B} = (B_x, B_y, B_z)$. Therefore, we have

$$u_x = -\frac{\omega\tau E_y}{B(1 + \omega^2\tau^2)}\left(-\omega_z\tau + \omega_x\omega_y\tau^2\right),$$

$$u_y = -\frac{\omega\tau E_y}{B(1 + \omega^2\tau^2)}\left(1 + \omega_y^2\tau^2\right),$$

$$u_z = -\frac{\omega\tau E_y}{B(1 + \omega^2\tau^2)}\left(\omega_x\tau + \omega_z\omega_y\tau^2\right) \tag{28}$$

---

[1]This differs from Blum and Rolandi's derivation, see [1]

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Note that in the case where $\vec{B} = (0, B_y, 0)$, the drift is *down* and the drift velocity $v_{d_0} = u_y = \omega\tau E_y/B_y$.

To correct for these distortions in MIPP, we work backwards from the pad plane, where the position measurement is made, and undo the drifting of the electron cloud. In steps of ds, the corrections may be derived from Eqns. 28 in the following manner:

$$u_{x_i} = \frac{dx_i}{dt} = \frac{dx_i}{dt}\frac{dt}{ds}v_d \tag{29}$$

where $x_i = x, y, z$ and $v_d^2 = u_x^2 + u_y^2 + u_z^2$. Therefore, at each step ds, the corrections are

$$
\begin{aligned}
dx &= ds\frac{\omega\tau E_y}{v_d B(1+\omega^2\tau^2)}\left(-b_z\omega\tau + b_x b_y\omega^2\tau^2\right), \\
dy &= ds\frac{\omega\tau E_y}{v_d B(1+\omega^2\tau^2)}\left(1 + b_y^2\omega^2\tau^2\right), \\
dz &= ds\frac{\omega\tau E_y}{v_d B(1+\omega^2\tau^2)}\left(b_x\omega\tau + b_z b_y\omega^2\tau^2\right)
\end{aligned}
\tag{30}
$$

where $b_i = B_i/|B|$, and all values are calculated at the current position.

5 TPC Track Reconstruction

5.1 Track Parameterization

The TPC track reconstruction algorithm used here is based on that used by the E910 collaboration. Ideally, a track in the TPC describes a helix:

$$y = y_0 + \frac{dy}{ds}s \tag{31}$$

$$x = x_0 + R\left[\sin\left(\phi_0 + \frac{hs}{R}\right) - \sin\phi_0\right] \tag{32}$$

$$z = z_0 + R\left[\cos\left(\phi_0 + \frac{hs}{R}\right) - \cos\phi_0\right] \tag{33}$$

where s is the distance travelled from the point (x_0, y_0, z_0), R is the radius of the circle the track forms in the bending plane, h is the helicity of the track, and $\phi_0 = \tan^{-1}(dx_0/dz_0)$.

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We can break the problem of track fitting into two parts since in the bending plane the track is a circle and in the $y - s$ plane the track is a straight line. Therefore, in the MIPP TPC bending $(x - z)$ plane, function we wish to minimize is

$$\chi^2_{bp} = \sum_i w_i \epsilon_i^2 \tag{34}$$

where "bp" stands for bending plane, w_i is the weight of the ith data point and in the case of MIPP,

$$\epsilon_i = \sqrt{(x_i - x_c)^2 + (z_i - z_c)^2} - R \tag{35}$$

In the MIPP non-bending plane, the function we wish to minimize is

$$\chi^2_{nbp} = \sum_i f_i((ks_i + y_0) - y_i)^2 \tag{36}$$

where f_i is the weight of the ith data point, s is the path-length traveled in the bending $(x - z)$ plane and $k = dy/ds$. We describe the method used to directly compute the track parameters.

5.1.1 Fits in the Bending Plane

Since we expect $\epsilon_i \ll R$, we find that to order $\mathcal{O}(\epsilon_i^2/2R)$

$$\epsilon_i = \frac{1}{2R}\left[\Delta x_i^2 + \Delta z_i^2 - R^2\right] \tag{37}$$

where $\Delta x_i = (x_i - x_c)$ and $\Delta z = (z_i - z_c)$.

The fit parameters here are x_c, z_c and R. However, in order to fit straight tracks (very large R), it is more convenient to fit for the radius of curvature, ρ. We therefore transform our coordinate system from Cartesian to cylindrical, as described in Fig.3:

$$z_i = r_i \cos \psi_i, \qquad x_i = r_i \sin \psi_i,$$
$$z_c = (R + d) \cos \phi, \quad x_c = (R + d) \sin \phi \tag{38}$$

Working out the algebra, Eq. 37 becomes

$$\epsilon_i = \frac{1}{2}\rho(r_i^2 + d^2) + d - r_i(1 + \rho d) \sin(\phi - \psi_i) \tag{39}$$
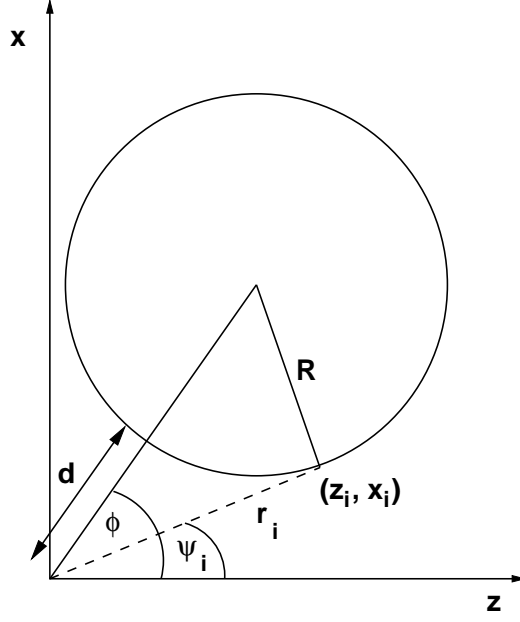
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Figure 3: Coordinate system used to fit the TPC data in the bending plane.

Factoring out the $(1 + \rho d)$ term, we find

$$\epsilon_i = (1 + \rho d)\eta_i \tag{40}$$

where

$$\eta_i = \kappa r_i^2 - r_i \sin(\phi - \psi_i) + \delta, \tag{41}$$

$$\kappa = \frac{1}{2}\left(\frac{\rho}{1 + \rho d}\right), \tag{42}$$

$$\delta = d\left(\frac{1 + \rho d/2}{1 + \rho d}\right) \tag{43}$$

Therefore, Eq. 34 becomes

$$\chi_{bp}^2 = (1 + \rho d)^2 \sum w_i \eta_i^2 \tag{44}$$

One nice feature of this expression for the function we wish to minimize is that one can always transform coordinates such that $\rho d \ll 1$, hence

$$\chi_{bp}^2 \simeq \sum w_i \eta_i^2 \tag{45}$$

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which is much more manageable than Eq. 44.

In [] it is shown that the χ^2_{bp} and three fit parameters ϕ, ρ, and d may be expressed using correlations:

$$C_{A,B} = \langle AB \rangle - \langle A \rangle \langle B \rangle \tag{46}$$

One finds

$$
\begin{aligned}
\phi &= \frac{1}{2} \arctan\left(\frac{2q_1}{q_2}\right), \\
\rho &= \frac{2\kappa}{\sqrt{1 - 4\delta\kappa}}, \\
d &= \frac{2\delta}{1 + \sqrt{1 - 4\delta\kappa}}
\end{aligned}
\tag{47}
$$

where

$$
\begin{aligned}
q_1 &= C_{r^2,r^2}C_{x,z} - C_{x,r^2}C_{z,r^2}, & (48) \\
q_2 &= C_{r^2,r^2}(C_{z,z} - C_{x,x}) - C_{z,r^2} + C_{x,r^2}, & (49) \\
\kappa &= \frac{C_{z,r^2}\sin\phi - C_{x,r^2}\cos\phi}{C_{r^2,r^2}}, & (50) \\
\delta &= -\kappa \langle r^2 \rangle + \langle z \rangle \sin\phi - \langle x \rangle \cos\phi & (51)
\end{aligned}
$$

Finally, one can also express χ^2_{bp} in terms of the correlations:

$$
\begin{aligned}
\chi^2_{bp} &= S_w(1 + \rho d)^2(C_{z,z}\sin^2\phi - 2C_{x,z}\sin\phi\cos\phi + \\
&\quad C_{x,x}\cos^2\phi - \kappa C_{r^2,r^2})
\end{aligned}
\tag{52}
$$

where $S_w = \sum w_i$

In practice, the track parameters d, ϕ, and ρ and fit χ^2 are computed directly using the equations above and maintaining the following symmetric matrix:

$$
\mathbf{A} = \begin{pmatrix}
w_i r_i^4 & w_i r_i^2 z_i & w_i r_i^2 x_i & w_i r_i^2 \\
 & w_i z_i^2 & w_i z_i x_i & w_i z_i \\
 & & w_i x_i^2 & w_i x_i \\
 & & & w_i
\end{pmatrix}
$$

where the index i implies the sum over all data points. Using the appropriate elements of this array, one can quickly and easily calculate the correlations necessary to determine the track parameters. Use of this matrix also makes adding new TPC hits to a fit track trivial.

5.1.2 Fits in the Non-bending Plane

Fits to the data in the $y - s$ plane is much simpler than the case for fits in the bending plane. Minimizing the function

$$\chi^2_{nbp} = \sum_i f_i((ks_i + y_0) - y_i)^2 \tag{53}$$

one finds that

$$k = \frac{\sum w_i s_i y_i \sum w_i - \sum w_i s_i \sum w_i y_i}{\sum w_i s_i^2 \sum w_i - (\sum w_i s_i)^2}, \tag{54}$$

$$y_0 = \frac{\sum w_i s_i^2 \sum w_i y_i - \sum w_i s_i y_i \sum w_i s_i}{\sum w_i s_i^2 \sum w_i - (\sum w_i s_i)^2} \tag{55}$$

and

$$\begin{aligned}\chi^2_{nbp} =\ & k \sum w_i s_i^2 + 2k y_0 \sum w_i s_i + y_0^2 \sum w_i - \\ & 2k \sum w_i y_i s_i - 2y_0 \sum w_i y_i + \sum w_i y_i^2 \end{aligned} \tag{56}$$

We therefore define the following matrix to be used in a similar manner as the matrix $\mathbf{A}$:

$$\mathbf{B} = \begin{pmatrix} w_i s_i^2 & w_i s_i & w_i s_i y_i \\ & w_i & w_i y_i \\ & & w_i y_i^2 \end{pmatrix}$$

The fit parameters k, y_0, and χ^2_{nbp} can be quickly calculated and updated as new hits are added to tracks using the appropriate elements of the matrix $\mathbf{B}$.

5.2 Track Finding Algorithm

5.3 Drift Velocity and Trigger Time Offset Calibration

6 TPC Vertex Reconstruction

7 TPC Particle ID
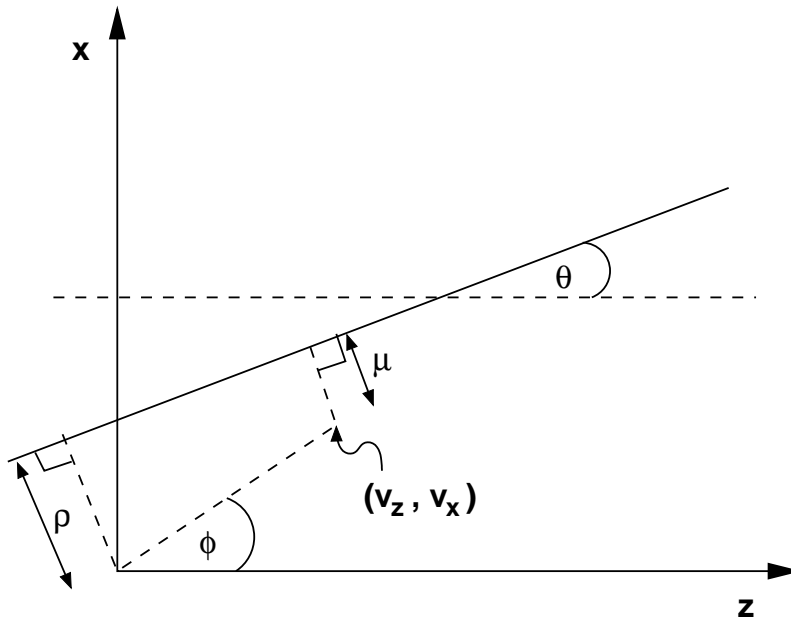
This is a work-in-progress...

Figure 4: Coordinate system used to fit TPC tracks to a vertex in a 2-d plane.

References

[1] Blum, W. and L. Rolandi, "Particle Detection with Drift Chambers", Springer-Verlag (19??), p. 50-51.